Anticipations, Recessions and Policy: An Intertemporal Disequilibrium Model [with Comments]
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Reviewed work(s):
Published by: L’INSEE / GENES on behalf of ADRES
Stable URL: http://www.jstor.org/stable/20076450
Accessed: 20/04/2012 13:35

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Anticipations, recessions and policy; an intertemporal disequilibrium model

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and Jeffrey SACHS *

This paper presents an intertemporal disequilibrium model with rational expectations, i.e. a model in which prices and wages may not adjust fast enough to maintain continuous market clearing. Therefore, optimizing firms and households base their intertemporal plans on anticipations of both future quantity constraints and future prices. Such a model shows clearly that the effect of a policy depends not only on its current values but its anticipated path. After a presentation of the model and its basic dynamics, we therefore consider the effects of various paths of fiscal policy on the economy.

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Both France and the United States have recently experienced major political changes. As a result, current economic policy is different from what it was, and even larger changes are anticipated in the coming years. Although the goal of those policies is to accomplish structural changes, stronger defense and less government intervention in the US, more equal income distribution and industrial reorganization in France, they will have and already are having macroeconomic effects on investment, consumption and employment. High anticipated deficits are blamed for the high long-term real interest rates in the US, anticipations of a higher fiscal burden on firms are blamed for the sluggishness of private investment in France.

The purpose of this paper is to present a model in which effects of anticipated as well as current changes in policy can be analyzed. Technically the model is an intertemporal disequilibrium model with rational expectations, i.e. a model in which agents take the anticipated future into consideration as rationally as they can, but where prices and wages may not adjust fast enough to maintain full employment. The paper therefore builds on two recent strands of research in macroeconomic fluctuations: rational intertemporal choice and disequilibrium analysis.

The first approach has emphasized that most decisions are intertemporal and thus depend as much on anticipated as on current prices. It has focused in particular on intertemporal substitution of consumption or leisure by households, on the optimal employment-investment decisions by firms (see for example, books by Barro [3], Lucas [18], Sargent [24]). This approach has led to a better understanding of the dynamic effects of either policies or real disturbances (in our own work for example, fiscal policy [2], oil price shocks [22]). Almost all of the work in this area has, however, maintained the assumption of market clearing, at least in the goods market (Hall [12] and Sachs [22] allow for a non-labor-market-clearing real wage).

The second approach, disequilibrium theory, has emphasized that, if prices are not fully flexible, most decisions must take into account not only prices but
quantity constraints. It has focused in particular on the implications for consumption and labor supply decisions by households, and for employment decisions of firms, giving a better foundation to many standard macroeconomic relations. It has shown how equilibria correspond to different regimes, each with distinct implications for policy. Although most of the work in this area has recognized the potential importance of anticipated future constraints (most notably Malinvaud [20]), it has usually not modeled behavior explicitly as intertemporal or considered the implications of rational expectations (an exception is Neary and Stiglitz [21]).

Can both approaches be combined? We believe that they can, in that agents attempt to make rational choices and to anticipate the future, even when some prices are not fully flexible. We realize that the assumption of rational expectations is overly strong and the lack of explicit foundations of price inertia is unsettling. Rational expectations appear however to be the most neutral way of allowing agents' decisions to depend on the future. We also believe that a model with more firmly grounded price inertia (possibly from desynchronization, such as in [25] and [8]) would lead to similar results, although we have not in this model attempted such an undertaking\(^1\).

The model we derive is slightly beyond analytical tractability. An analytical treatment can only be offered by taking shortcuts, something we have explored in [6], [9]. The choice here has been to solve the complete model by numerical simulations. Given that the model is derived from maximizing behavior and that its size is small enough, results can easily be traced to specific assumptions about parameters and policy.

The paper is organized as follows: Section 1 characterizes households' and firms' behavior, as well as the intertemporal equilibrium. In particular, it clarifies the role of short and long interest rates in investment and consumption decisions, and the relation between market value, profit, profitability, the real wage and investment. It shows how the expectation of future constraints may lead to anticipatory buying by consumers and firms. Section 2 displays the basic dynamic mechanisms of the model, through a focus on the following questions: What is the role of investment, both through demand and supply, in the transmission of shocks, a question stressed by Malinvaud [20]? Can sharp deflations in response to lower money growth be destabilizing, as suggested by Keynes and
more recently by Tobin [26]? Is the responsiveness of real wages to unemployment stabilizing or destabilizing, a question raised throughout the research on disequilibrium (starting with Barro-Grossman [4])? Section 3 returns to one of the policy issues relevant in France today: What happens if firms anticipate an increase in their fiscal burden in the future? No attempt is made to fit specific facts and magnitudes, our intention being to clarify the various economic forces set in play by such anticipations. More generally, our focus in most of the paper is as much on methodology as on substantive economic issues. Carefully calibrated simulation models will be necessary to reach firm answers to many of the issues raised in the paper.

1. We believe that models of price inertia will lead to price structures in which relative prices, except for the real wage, are approximately correct, but in which their average level, the price level, does not adjust quickly. As this paper relies on price level and real wage inertia only (and so implicitly assumes all other relative prices to be flexible), we feel that results would not be drastically changed by a more explicit derivation of price behavior.
1 The model

1.1. General description

Our choice has been to build the simplest model in which households and firms have nontrivial intertemporal choices. Thus, we assume that the economy is closed and that there is one produced good, used either for consumption or investment. There are three tradable assets, money, debt and equities.

Households and firms take as given both current and anticipated prices and quantity constraints. Although the future may be uncertain, they act as if they know it with certainty. Households are all identical and maximize the discounted sum of utility; their problem is intertemporal as they have to choose between consumption now and consumption later. Firms are identical and maximize their value, which is the discounted sum of anticipated cash flows. Labor is a variable factor but capital is quasi-fixed: changes in capital are costly, so that the investment decision presents also an intertemporal problem.

The solutions to the maximization problems of households and firms give a set of actual demands (or supplies) which satisfy both the budget constraint and (current and anticipated) quantity constraints. (These are, in the disequilibrium terminology, “DREZE” demands [11].) We can also, for both maximization problems, find for each constraint the lowest value of this constraint such that it is not binding. We shall refer to this set of values as the set of shadow demands (these correspond to “BENASSY” demands). It follows that each actual demand can be expressed as the minimum of the constraint and the shadow demand.

Each market has rationing rules governing the allocation of goods to the constrained side; we allow these rules to depend on shadow demands. Market clearing requires that two conditions be satisfied: the first is that at most one side of the market be constrained, the second that actual demand and actual supply be equal. We follow others in this field by allowing asset prices to adjust and thus asset markets to clear. We do not however follow standard usage in our treatment of the labor market: we assume that households supply all the labor demanded by firms and that firms are therefore never constrained in the labor market. By doing so, we eliminate the regime of “repressed inflation” (following MALINVAUD’S terminology [19]) and are left with only two regimes, a “Keynesian regime” when suppliers of goods

---

2. This strong assumption allows us to formalize agents and firms as solving certainty problems. While convenient, and probably necessary at this stage, it does not allow us to look at the effects of uncertainty per se on behavior.
are constrained and a "classical regime" when buyers of goods are constrained. We feel that little is lost by this simplification, at least for the experiments we consider, and that the benefit in increased simplicity is substantial.

Prices adjust over time as functions, not of excess actual demands which are, by construction, identically zero, but of excess shadow demands.

At any time $t$, an intertemporal equilibrium is a sequence of shadow demands, a sequence of actual demands and a sequence of prices, consistent with maximizing behavior and the rationing rules, such that both current and future markets are anticipated to clear. If exogenous variables take over time their anticipated values, the intertemporal equilibrium is actually realized over time. If at some time $t+\tau$, there are unanticipated changes in current or anticipated exogenous variables, a new intertemporal equilibrium for period $t+\tau$ and future periods must be recomputed.

We now describe the model in detail.

1.2. The behavior of firms

All firms are identical and we shall not distinguish between individual and aggregate values. The time index $t$ will also be deleted, whenever convenient.

The technology of the firm is characterized by:

a) A production function, with constant returns to scale:

$$Q = F(K, L)$$

b) An installation function, giving the number of goods used up in installation of I units of investment:

$$I \Phi(I/K) \quad \Phi(0) = 0, \quad \Phi'(\cdot) > 0$$

The total number of goods needed to invest at rate I is therefore:

$$J = I + I \Phi(I/K) = I (1 + \Phi(I/K))$$

c) An accumulation equation, with exponential depreciation at rate $\mu$:

$$\dot{K} = I - \mu K$$

The firm takes as given the sequence of real wages and real interest rates $|W/P, r_t|_{t=0,\ldots,\infty}$ and the sequence of constraints on the amount of goods it can sell and the amount of goods in can invest, $|Q_t, I_t|_{t=0,\ldots,\infty}$. By assumption, there are no constraints on the amount of labor it can buy. The firm maximizes its value, which is the present value of cash flows:

$$V = \int_0^{\infty} (Q - (W/P) L - J) e^{-\int_0^t r_s ds} dt$$

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Let \( e^{-\int_0^t r_s ds} q_t \) be the costate variable associated with the accumulation equation, and \( \lambda_Q, \lambda_I \) the Lagrange multipliers associated with the quantity constraints; the Hamiltonian is therefore:

\[
H = e^{-\int_0^t r_s ds} \left[ F(K, L) - (W/P) L - I (1 + \Phi (I/K)) + q (I - \mu K) + \lambda_Q (Q - F(K, L)) + \lambda_I (I - I) \right]
\]

Necessary conditions for maximization are:

\[
F_L(K, L) (1 - \lambda_Q) = W/P \quad ; \quad \lambda_Q (Q - Q) = 0 \quad ; \quad \lambda_Q \geq 0
\]

\[
1 + (I/K) \Phi' (I/K) + \Phi (I/K) = q - \lambda_I \quad ; \quad \lambda_I (I - I) = 0 \quad ; \quad \lambda_I \geq 0
\]

\[
\dot{q} = (r + \mu) q - (1 - \lambda_Q) F_K (K, L) - (I/K) F' (I/K)
\]

\[
\lim_{t \to \infty} e^{-\int_0^t r_s ds} q_t K_t = 0
\]

They can be rewritten more intuitively as follows;

Define shadow labor demand and quantity supply \( L^d, Q^s \) s.t.:

1. \( L^d, Q^s \mid F_L(L^d, K) = W/P \quad ; \quad Q^s = F(K, L^d) \)

2. Then \( Q = \min (Q^s, \bar{Q}) \quad ; \quad L \mid F(K, L) = Q \)

Define shadow investment and investment spending, \( I^d, J^d \) s.t.:

3. \( I^d, J^d \mid 1 + \Phi (I^d/K) + (I^d/K) \Phi' (I^d/K) = q \quad ; \quad J^d = I^d (1 + \Phi (I^d/K)) \)

4. Then \( I = \min (I^d, I) \quad ; \quad J = \min (J^d, J) \quad , \quad J = I (1 + \Phi (I/K)) \) and:

5. \( \dot{q} = (r + \mu) q - (W/P) (F_K (K, L)/F_L (K, L)) - (I/K)^2 \Phi' (I/K) \)

6. \( \lim e^{-\int_0^t r_s ds} q_t K_t = 0 \)

Given the capital stock, the real wage and the possibly binding output constraint, the employment and output decisions of the firm are straightforward. The investment decision is of more interest and is characterized by equations (3) through (6): To see what they imply, we can rewrite them further. Inverting (3), and integrating (5) forward subject to the transversality condition (6) gives:

7. \( I = \min (I^d, I) \quad ; \quad I^d/K = H(q) \quad ; \quad H(1) = 0 \quad ; \quad H'(...) > 0 \)

8. \( q = \int_0^\infty [(W/P) (F_K (K, L)/F_L (K, L)) + (I/K)^2 \Phi' (I/K)] e^{-\int_0^t (r_s + \mu) ds} dt \)

3. Costs of adjustment are a simple way of deriving a well-behaved flow investment demand. The specific functional form is a special case of Lucas [17] which preserves CRTS of technology with respect to \( K, L, I \). This subsection builds heavily on Abel [1], Hayashi [13] and Blanchard [9].

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Investment is the minimum of shadow investment and the constraint. Shadow investment in turn depends on \( q \), the present value of marginal profits, usually called Tobin’s \( q \). Marginal profit is the sum of two terms; the second is the reduction in the cost of installation made possible by an additional unit of capital and is a minor factor in profits. The first is more interesting and depends very much on the regime.

In the classical regime, i.e. if the firm is not output constrained, the marginal product of labor equals the wage and this first term is simply the marginal product of capital. Furthermore, if the firm does not anticipate to be ever output constrained in the future, a particularly nice result arises: the shadow price \( q \) is equal to the observable average value of capital \( V/K \) (Hayashi [13]). In the absence of constraints on investment spending, there is then a direct relation between the firm’s value and its investment behavior. An increase in the real wage for example decreases employment, the marginal product of capital, and thus investment and the value of the firm.

In the Keynesian regime, the firm is output constrained. The first term then is the marginal wage savings, i.e. the real wage times the marginal rate of substitution. There is no longer a close relation between \( q \) and \( V/K \), between investment and the value of the firm. The effect of an increase in the real wage is now to increase the marginal wage savings and to increase \( q \) and investment as capital becomes more attractive than labor to produce the same level of output. Although it now increases investment, the increase in the wage still decreases profit and thus the value of the firm. An increase in output increases both marginal and average profit and thus both \( q \) and \( V/K \). The effect on marginal profit however depends on the elasticity of substitution while the effect on average profit does not.

Finally, the effects of a constraint on investment demand are easily characterized. Anticipated constraints on investment demand lead, *ceteris paribus*, to an anticipated lower capital accumulation, thus to higher marginal profits, to a higher \( q \) and higher investment demand today. It therefore generates anticipatory buying of investment goods. The effect is represented graphically in figure 1.

Since asset markets are assumed to clear and the financial structure of firms is thus irrelevant for firms’ or agents’ decisions, the following structure is convenient: Firms have real debt \( B \) outstanding, paying the real interest rate. They finance all investment from retained earnings and do not issue new debt or new equity. The assumption that there is real debt outstanding implies that there is an observable interest rate but is otherwise irrelevant. (The assumption that all investment is internally financed implies that in equilibrium personal savings are zero.) The amount of profits paid to equity owners is as a result:

\[
\pi = (Q - (W/P)) L - J - rB
\]
1.3. The behavior of consumers-workers

All consumers are identical and we shall not distinguish between individual and aggregate values.

Consumers derive utility from consumption and real money balances. Leisure does not explicitly enter the utility function; desired labor supply is $L^*$. Households take as given the sequence of prices, real wages, real interest rates and profits paid to equity owners, as well as the sequence of labor they supply and the amount of goods they can buy $\{L_t, C_t\}_{t=0}^{\infty}$. They maximize the present value of utility:

$$U = \int_0^\infty u(C_t, M_t/P) e^{-\delta t} dt$$

Defining $A = B + M_t/P$, the budget constraint can be written as:

$$A = rA + (W_t/P) L + \pi - [(r + \Pi_t/P) (M_t/P) - C]$$

4. Considering the effects of a change in output only makes sense for an output constrained firm. The « neoclassical » approach is slightly confusing in this respect when it treats output as given, but maintains the assumption that the wage equals the marginal product of labor.

5. Around $W_t/P = F_L(K, L)$ and for $L$ given by $Q = F(K, L)$ the effect of a change in output on marginal profit is given by:

$$\frac{d}{dQ} [(W_t/P) (F_K(K, L)/F_L(K, L)))] = (F_K(K, L)/F_L(K, L)) \sigma_{KL}$$

where $\sigma_{KL}$ is the elasticity of substitution between $K$ and $L$. Thus, the smaller $\sigma_{KL}$, the larger the effects of a decrease in output on marginal profit.
Define the costate variable associated with the budget constraint as \( e^{-\delta t}x_t \), and the Lagrange multiplier associated with the quantity constraint on consumption as \( \lambda_C \). The Hamiltonian is then:

\[
H = e^{-\delta t} [u(C, M/P) + \lambda_C (\bar{C} - C) + x (rA + (W/P) \bar{L} + \pi - (r + \bar{P}/P) M/P - C)]
\]

Necessary conditions for maximization are:

\[
\begin{align*}
  u_C(C, M/P) &= x + \lambda_C; \quad \lambda_C (\bar{C} - C) = 0; \quad \lambda_C \geq 0 \\
  u_m(C, M/P) &= x (r + \bar{P}/P) \\
  \dot{x} &= (\delta - r) x \\
  \lim_{t \to \infty} e^{-\delta t} x_t &= 0
\end{align*}
\]

They can be rewritten more intuitively as follows:

Define shadow consumption demand \( C^d \) such that:

\[
\begin{align*}
  (10) & \quad C^d | u_C(C^d, M/P) = x \\
  (11) & \quad \text{Then } C = \min(C^d, \bar{C}) \\
  (12) & \quad u_m(C, M/P) = x (r + \bar{P}/P) \\
  (13) & \quad \dot{x} = (\delta - r) x; \quad \lim_{t \to \infty} e^{-\delta t} x_t = 0
\end{align*}
\]

In the absence of constraints on consumption, consumers equalize the marginal rate of substitution between money balances and consumption to the nominal interest rate. The path of consumption is determined by (13) which gives the behavior of marginal utility, and the budget constraint. Approximately, (13) gives the shape of the path and the budget constraint the highest feasible level of this path.

Current constraints on consumption lead to forced savings and more consumption later. Anticipated constraints have the same effect on consumption as on investment. They lead to anticipated forced savings, thus to a higher feasible level of consumption today. The effect is represented graphically in figure 2 when \( \delta = r \) and \( M/P \) is constant, so that agents choose the highest feasible constant level of consumption.

We shall introduce the government later.

1.4. Equilibrium given prices and wages

Equilibrium in the goods market, requires that actual supply equals actual demand. It also requires that at most one side be constrained. From (2), actual aggregate supply is

\[
Q = \min(Q^s, \bar{Q})
\]
The present value of $A_0$, $A_2$ areas equals the present value of $A_1$. Anticipatory consumption spending path if constraint $C$ anticipated at $t$ for $(t_1, t_2)$.
From (4) and (11), actual aggregate demand is

\[ Q = \min (Q^d, \bar{Q}) \quad \text{where} \quad Q^d = C^d + J^d, \quad \bar{Q} = \bar{C} + \bar{J} \]

The above conditions for equilibrium imply:

\[ \bar{Q} = \min (Q^s, Q^d) \]

Rationing rules are as follows: If supply is constrained, there is uniform rationing of firms. If demand is constrained, consumption and investment are rationed according to:

\[ \bar{C} = C^d - a (Q^d - \bar{Q}) \quad ; \quad a \in [0, 1] \]
\[ \bar{J} = J^d - (1 - a) (Q^d - \bar{Q}) \quad ; \quad I \mid I(1 + \Phi (I/K)) = J \]

Thus, in general, rationing depends on shadow aggregate demand. If \( a \) equals 0 or 1, however, there is rationing of investment only or consumption only.

In the labor market, we have assumed that firms can always hire the labor they demand. Thus, labor is given by:

\[ L \mid F (L, K) = \bar{Q} \]

1.5. Movement of prices and wages

Although there is by construction zero excess actual demand in all markets, shadow excess demands may be different from zero. Thus a natural assumption is to allow prices and wages to respond to these excess demands over time.\(^6\) They are assumed to follow:

\[ \frac{\hat{P}}{P} = \beta (Q^d - Q^d) \]
\[ \frac{\hat{W}}{W} = \theta (L - L^*) + \sigma \frac{\hat{P}}{P} \]

\( \beta, \theta \) measure the response of prices and wages to goods and labor market conditions. \( \sigma \) measures the degree of indexing of wages.

1.6. Intertemporal equilibrium

The set of equations is reorganized and presented in table 1. Equations (1.1) to (1.6) give shadow demands and supplies. These depend on three sets of variables, policy variables (M), state variables given from the past...

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6. An alternative formalization would be to solve for flexible prices and wages and assume partial adjustment of actual prices and wages. In the absence of more explicit assumptions about the source of price and wage inertia, it is difficult to decide which formalization is better.
(P, W, K) and costate variables which depend on the anticipated future (q, x). Equations (1.7) to (1.11) show how actual demands and supplies follow from shadow demands and supplies.

Equation (1.12) determines the real interest rate from equality of money demand and money supply. Equations (1.13) and (1.14) give the equations of motion of the costate variables (q, x), equations (1.16) to (1.18) the equations of motion of the state variables (P, W, K). Finally, equation (1.19) gives the behavior of V, which does not affect any other variable in the model.

Table 1

The complete model: equations of motion

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>(1.1)</td>
<td>( L^d</td>
</tr>
<tr>
<td>(1.2)</td>
<td>( Q^s</td>
</tr>
<tr>
<td>(1.3)</td>
<td>( I^d</td>
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<tr>
<td>(1.4)</td>
<td>( J^d</td>
</tr>
<tr>
<td>(1.5)</td>
<td>( C^d</td>
</tr>
<tr>
<td>(1.6)</td>
<td>( Q^d</td>
</tr>
<tr>
<td>(1.7)</td>
<td>( \bar{Q} = \min (Q^d, Q^s) )</td>
</tr>
<tr>
<td>(1.8)</td>
<td>( L</td>
</tr>
<tr>
<td>(1.9)</td>
<td>( \bar{C}</td>
</tr>
<tr>
<td>(1.10)</td>
<td>( J</td>
</tr>
<tr>
<td>(1.11)</td>
<td>( I</td>
</tr>
<tr>
<td>(1.12)</td>
<td>( u_m(\bar{C}, M/P) = x (r + \bar{P}/P) )</td>
</tr>
<tr>
<td>(1.13)</td>
<td>( \dot{q} = (r + \mu) q - (W/P) (F_K(K, L)/F_L(K, L)) - (I/K)^\Phi'(I/K) )</td>
</tr>
<tr>
<td>(1.14)</td>
<td>( \dot{x} = (\delta - r) x )</td>
</tr>
<tr>
<td>(1.15)</td>
<td>( \lim_{t \to \infty} x_t e^{-\delta t} = 0 ); ( \lim_{t \to \infty} K_t q_t e^{-\int_0^t r_s ds} = 0 )</td>
</tr>
<tr>
<td>(1.16)</td>
<td>( \dot{\bar{P}}/P = \beta (Q^d - Q^s) )</td>
</tr>
<tr>
<td>(1.17)</td>
<td>( \dot{W}/W = \theta (L - L^*) + \sigma \dot{\bar{P}}/P )</td>
</tr>
<tr>
<td>(1.18)</td>
<td>( \dot{\bar{K}} = I - \mu K )</td>
</tr>
<tr>
<td>(1.19)</td>
<td>( \dot{\bar{V}} = r V - [(\bar{Q} - (W/P) L) - J] )</td>
</tr>
</tbody>
</table>

At any point of time, an intertemporal equilibrium is a sequence of quantities \((L^d, Q^s, I^d, C^d, Q^d, L, Q, I, J, \bar{C}, K)\) and a sequence of prices \((r, q, x, P, W, V)\) which, given the sequence of policy \((M)\) and initial conditions \((P_0, W_0, K_0)\) satisfy equations (1.1) to (1.9) for the current and all future periods.

ANTICIPATIONS AND DISEQUILIBRIUM
1.7. Steady state

Before we turn to the dynamics, we briefly characterize the steady state of the model, when $\dot{P} = \dot{W} = \dot{K} = \dot{x} = \dot{q} = 0$. We denote steady state values by stars. Intertemporal utility maximization implies, from (1.14), that $r^* = \delta$. The interest rate is always equal to the subjective discount rate in steady state; equivalently the long-run elasticity of savings at $\delta$ is infinite. From (1.3) and (1.8), as $I = I^4$, $q$ must be sufficient to generate gross investment equal to depreciation:

$$q^* = 1 + \Phi (\mu) + \mu \Phi' (\mu) > 1$$

In turn, (1.13) determines the marginal product of capital and thus the steady state level of capital stock:

$$K^* \left| F_K(K^*, L^*) + \mu^2 \Phi'(\mu) = (\delta + \mu) q^* \right.$$  

If $\mu = 0$, so that $q = 1$, this condition reduces to the familiar condition:

$$F_K(K^*, L^*) = \delta$$

Consumption is determined by:

$$C^* = F(K^*, L^*) - \mu K^* (1 + \Phi (\mu))$$

Finally, the level of real money balances is given by:

$$u_m(C^*, M/P^*)/u_C(C^*, M/P^*) = \delta$$

Money is clearly neutral in the long run in this model.

2 Comparative dynamics

To show the basic dynamics of the model, this section concentrates on the dual role of investment as a determinant of demand and later of supply through capital accumulation, and on the stabilizing or destabilizing role of prices and wages.

2.1. Functional forms and parametrization

To simulate the model, specific functional forms must be chosen for utility and technology, and numerical values must be chosen for the parameters. These assumptions are summarized in Table 2.
Table 2

Functional forms, parameters and steady state values

Consumers

\[ u = (\xi C^{-\rho_1} + (1 - \xi) (M/P)^{-\rho_1})^{-1/\rho_1} \]

\[ U = \int_0^\infty (\ln u_t) e^{-\delta t} \, dt \]

\[ \xi = 0.95 \quad ; \quad \rho_1 = 6 \Rightarrow \sigma_{Gm} = 1/(1 + \rho_1) = 0.14 \]

\[ \delta = 0.03 \]

Firms

\[ Q = \gamma (aK^{-\rho_2} + (1 - a) L^{-\rho_2})^{-1/\rho_2} \]

\[ \varphi (I/K) = b. (I/K) \quad ; \quad \dot{K} = I - \mu K \]

\[ \gamma = 0.25 \quad ; \quad a = 0.25 \quad ; \quad \rho_2 = 1 \Rightarrow \sigma_{KL} = 1/(1 + \rho_2) = 0.5 \]

\[ b = 4 \quad ; \quad \mu = 0.025 \]

Rationing rules and price adjustment

\[ \beta = 0.03 \quad ; \quad \theta = 0.015 \quad ; \quad \sigma = 1 \]

\[ a = 0.9 \]

Exogenous variables

\[ L^* = 8 \quad ; \quad M = 4 \]

Implied steady state values

\[ Q^* = 2 \quad (W/P)^* L^* = 1.50 \quad r^* K^* = 0.25 \]

\[ C^* = 1.78 \quad J^* = 0.22 \quad I^* = 0.20 \]

\[ r^* = 0.03 \quad P^* = 2 \quad W^* = 0.375 \]

\[ q^* = 1.2 \quad V^* = 9.60 \quad K^* = 8 \]

Instantaneous utility is CES in consumption and money balances. The elasticity of substitution between utility in different periods is assumed to be unity, so that cardinal utility is logarithmic. (Under uncertainty, this last assumption implies unit constant relative risk aversion.)

The production function is also CES in capital and labor. The installation function \( \varphi \) (I/K) is linear in I/K, so that total cost of installation is quadratic in I.
The unit period is a quarter. Thus all flow variables and parameters with a time dimension are at quarterly rates. There is no attempt to calibrate the model to fit a particular economy; parameter values are chosen to either be reasonable, or to fit existing empirical evidence or to have reasonable implications. Implied steady state values for the main variables are given also in Table 2. A few parameters require justification.

The elasticity of substitution between consumption and real money balances is also the interest elasticity of money demand. It is chosen to be 0.14, which corresponds to empirical estimates of this long-run elasticity. The subjective discount rate $\delta$ implies a steady-state annual interest rate of 12%, which is roughly in line with the average profit rate on corporate capital in the U.S.

The convexity coefficient for installation costs, $b$, implies that a ratio of annual investment to capital of 10% leads to average installation costs equal to 10% of the purchase price of capital, and marginal installation costs equal to 20% of purchase price. The elasticity of substitution $\sigma_{KL}$ has been chosen to be relatively low, 0.5, to reduce the scope for substitution of capital and labor in response to short-run changes in factor prices.

The price and wage adjustment parameters will be discussed later; they clearly do not affect the steady state. The proportion of rationing allocated to consumption, $a$, is chosen to equal approximately its share in aggregate demand, 0.9.

### 2.2. Method of solution

Dynamic simulations present two problems usually not encountered in macroeconomic simulations.

The first is standard in rational expectations models. The initial values of $q$, $x$ and $V$ are not given from the past but determined from the requirement that the transversality conditions be satisfied. A dynamic simulation is thus a two-point boundary value problem, with initial conditions for $K, P, W$ and terminal conditions for $q, x$ and $V$, (i.e. the transversality conditions). The technical method of solution is that of multiple shooting (see [16] for details).

The second is specific to disequilibrium models and comes from the presence of minimum functions. We replace the minimum function ([1.7] in Table 1) by a CES function with low elasticity. In practice, an elasticity of 0.005 is enough to replicate the minimum rule.

We now turn to the simulations. The first two focus on the behavior of quantities, and to do so, assume fixed prices and wages ($\beta = \delta = 0$).

### 2.3. Simulation 1. Demand shocks and Keynesian unemployment

Suppose agents decide to save more in order to consume more later, i.e. that there is a temporary decrease in the subjective discount rate. More
precisely, assume that $\delta$ decreases unanticipatedly at time 0 and thereafter follows:

$$\delta = 0.03 + 0.9(\delta - 0.03) \quad ; \quad \delta_0 = 0.025$$

The effects are summarized in Table 3, for three different sets of values of $\sigma_{cm}$ and $\sigma_{KL}$.

**Table 3**

*A temporary decrease in the discount rate; P, W fixed*

<table>
<thead>
<tr>
<th>Quarter.....</th>
<th>Reference case</th>
<th>High $\sigma_{cm}$ (Flat LM)</th>
<th>Low $\sigma_{KL}$ (Steep IS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(\sigma_{cm}=0.14; \sigma_{KL}=0.5)$</td>
<td>$(\sigma_{cm}=2; \sigma_{KL}=0.5)$</td>
<td>$(\sigma_{cm}=2; \sigma_{KL}=0.1)$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>Variable</td>
<td>(c.............</td>
<td>$-2.6$</td>
<td>$-1.6$</td>
</tr>
<tr>
<td></td>
<td>$q.............</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>$QD............</td>
<td>$-1.9$</td>
<td>$-1.6$</td>
</tr>
<tr>
<td></td>
<td>$QS............</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>$r............</td>
<td>$-2$</td>
<td>$-1.2$</td>
</tr>
<tr>
<td></td>
<td>$V/K...........</td>
<td>3.6</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>$u............</td>
<td>2</td>
<td>1.7</td>
</tr>
</tbody>
</table>

All variables are in % deviation from steady state except.

$u$: unemployment rate, measured in %.

$r$: absolute deviation from steady state, measured in % at annual rate.

The central role is played by investment. Lower aggregate demand lowers output and this in turn has two effects: it lowers the demand for money and thus the sequence of anticipated interest rates; it lowers the sequence of anticipated marginal profits. Which of the two lower sequences dominates and whether $q$ and investment go up or down depends crucially on $\sigma_{cm}$ and $\sigma_{KL}$.

7. The time-separable utility function we use implies the same short- and long-run elasticities of money demand both with respect to consumption and interest rates.

8. Actual estimates of this installation cost coefficient, $b$, derived from regressions of investment on market value, are much higher. They are however implausibly high and likely to be biased upwards (see [7]).

9. Checking global stability before proceeding with simulations would be desirable but is impossible. Checking local stability if feasible. Stability conditions would combine the results of Blanchard and Kahn [10] for linear systems with rational expectations and the results of Ito [14] for systems with regimes and possibly discontinuous derivatives. We have not checked them but have not encountered problems of convergence in simulation.
If $\sigma_{Gm}$ is large, the demand for money is very interest elastic, interest rates decrease little and $q$ goes down. If $\sigma_{KL}$ is very small, the decrease in output and employment reduces strongly marginal profit $^{10}$ and $q$ also goes down. These two cases are shown in columns 4 to 6 and 7 to 9 respectively.

The response of investment in turn, through a multiplier effect, decreases consumption. The larger and more prolonged the decrease in investment, the larger the initial decline in consumption and thus the overall recession. The similarity of our results to the simple ISLM is striking: the impact effect is larger, the flatter the LM (the larger $\sigma_{Gm}$), the steeper the IS (the smaller $\sigma_{KL}$). As the recession slowly ends, net investment becomes positive again. In none of the three cases does the economy experience a supply constraint; it always remains in Keynesian unemployment.

Table 3 also shows clearly the different behavior of marginal $q$ which affects investment and is unobservable, and average $q$ which might be observable through the stock market valuation of firms. Although $q$ may go down, $V/K$ goes up in all three cases: the effect of lower real rates dominates the effect of lower average profit. Thus, the stock market goes up while output and possibly investment go down.

2.4. Simulation 2. Supply shocks and classical unemployment

Suppose that the economy is affected by an adverse, unanticipated technological shock which disappears over time. More precisely, assume that $\gamma$ follows after $t_0$:

$$(\gamma - 0.25) = 0.9 (\gamma_{-1} - 0.25) \quad ; \quad \gamma_0 = 0.2375$$

This decrease of productivity of 5 % initially is in the spirit of a temporary increase of the price of oil (see [22] for a specific treatment of such an increase with a more adequate treatment of technology). The effects are summarized in Table 4, again for three sets of values of $\sigma_{Gm}$ and $\sigma_{KL}$.

The central role is again played by investment, this time not as a component of demand but as a determinant of supply through capital accumulation. The impact effect on production however depends only on the technology: after the decrease in productivity, employment must decrease until the marginal product of labor is again equal to the unchanged real wage. The size of the adjustment depends on $\sigma_{KL}$. If $\sigma_{KL}$ is low, the required decrease in employment is small; if $\sigma_{KL}$ is larger, the decrease is larger. The impact effect on the marginal product is independent of $\sigma_{KL}$ to the first order and proportional to the share of capital in output.

The adjustment process and the length of the period of classical unemployment depends however very much on investment. Two effects are again present: the first is a lower sequence of real interest rates, the second is a lower sequence of marginal products. Whether the first or the second dominates depends again on the interest elasticity of money demand.

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### Table 4

*A temporary decrease in productivity; \( P, W \) fixed*

<table>
<thead>
<tr>
<th>Quarter Variable</th>
<th>Reference case ((\sigma_\text{CM} = 0.14; \sigma_\text{KL} = 0.5))</th>
<th>High (\sigma_\text{CM}) ((\sigma_\text{CM} = 2; \sigma_\text{KL} = 0.5))</th>
<th>Low (\sigma_\text{KL}) ((\sigma_\text{CM} = 0.14; \sigma_\text{KL} = 0.1))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3</td>
<td>4 5 6</td>
<td>7 8 9</td>
</tr>
<tr>
<td>C……………</td>
<td>0 4 12</td>
<td>0 4 12</td>
<td>0 4 12</td>
</tr>
<tr>
<td>(q)……………</td>
<td>-14.2 - 7.6 - 1.7</td>
<td>-7.4 - 7.4 - 7.0</td>
<td>-6.9 - 3.9 - 0.7</td>
</tr>
<tr>
<td>(\text{QD})……</td>
<td>5.9 2.7 0.0</td>
<td>-5.5 - 3.1 - 0.5</td>
<td>2.7 1.5 0.0</td>
</tr>
<tr>
<td>(\text{QS})……</td>
<td>15.2 8.2 1.6</td>
<td>-5.5 - 4.0 - 2.0</td>
<td>7.6 3.9 0.5</td>
</tr>
<tr>
<td>(K)……………</td>
<td>-10.7 - 6.4 - 0.2</td>
<td>-10.7 - 9.4 - 7.4</td>
<td>-5.8 - 3.3 - 0.9</td>
</tr>
<tr>
<td>(r)……………</td>
<td>0.0 1.0 1.1</td>
<td>0.0 - 2.8 - 4.9</td>
<td>0.0 0.5 0.7</td>
</tr>
<tr>
<td>(u)……………</td>
<td>-7.7 - 5.0 - 1.4</td>
<td>0.0 0.0 0.0</td>
<td>-4.6 - 2.8 - 0.5</td>
</tr>
<tr>
<td></td>
<td>8.2 4.5 1.0</td>
<td>8.1 7.4 6.2</td>
<td>1.3 0.2 - 0.6</td>
</tr>
</tbody>
</table>

See Table 3 for definitions.

Columns 4 to 6 show how, if interest rates do not adjust, investment falls and the recession is substantially deeper and longer.

What happens to shadow aggregate demand in the process of adjustment is ambiguous. Although both \(q\) and wealth may go down, the anticipation of constraints on both consumption and investment spending may lead both consumers and firms to attempt anticipatory buying. The anticipatory buying effect dominates in the first and third cases in Table 4, the lower wealth and \(q\) effect dominates in the second case. In all cases, however, shadow aggregate demand is less than supply and both firms and consumers are rationed in their purchases. Thus investment is less than the value implied by \(q\).

We now turn to the effects of price-wage dynamics on the process of adjustment. For this, we shall consider the effects of an unanticipated decrease in nominal money by 5% which is assumed by agents to be permanent. The reference values of \(\beta\), \(\sigma\), \(\theta\) are 0.03, 1, 0.015 respectively. The value of \(\beta\) implies that an excess shadow demand of 10% increases prices by 2.4% over a period of a year\(^{11}\). The value of \(\sigma\) implies complete indexing of

\[\frac{d}{dQ} \left[ (\ln W/P)(F_K(K, L)/F_L(K, L)) = (F_K(K, L)/F_L(K, L)) \sigma_{KL} \right] \]

where \(\sigma_{KL}\) is the elasticity of substitution between \(K\) and \(L\). Thus, the smaller \(\sigma_{KL}\), the larger the effects of a decrease in output on marginal profit.

10. Around \(W/P = F_L(K, L)\) and for \(L\) given by \(Q = F(K, L)\) the effect of a change in output on marginal profit is given by:

11. There is inflation therefore only if there is excess demand for goods. This assumption is acceptable only because we have assumed a zero rate of growth of money. If this rate of growth were positive, the price equations would have to be modified.
wages and the value of \( \theta \) implies that 5% unemployment in excess of the natural rate (zero in our model) decreases real wages by 2.4% over a period of a year.

### 2.5. Simulation 3. The adjustment of prices and the stabilizing Mundell effect

Tobin [26] recently formalized an argument of Keynes that a fast adjustment of prices may lead to a larger recession in response to a decrease in money growth: if prices adjust fast, there will be a large decrease in inflation and expected inflation. Thus real rates will increase both because of higher nominal rates and because of lower expected inflation; this second channel is usually referred to as the Mundell effect. Thus the faster the prices adjust, the larger the Mundell effect, the higher the real rate and the larger, Tobin suggests, the effect on aggregate demand.13

We now check in our model the effects of the speed of adjustment of prices on the impact of the decrease in money and the length of the recession. The results are given in Table 5 for three values of \( \beta \), 0.01, 0.03, 0.05.

**Table 5.**

The adjustment of prices and the Mundell effect

The effects of a decrease in money

<table>
<thead>
<tr>
<th>Variable</th>
<th>Reference case (( \beta = 0.03 ))</th>
<th>Slow adjustment (( \beta = 0.01 ))</th>
<th>Fast adjustment (( \beta = 0.05 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3</td>
<td>4 5 6</td>
<td>7 8 9</td>
</tr>
<tr>
<td>Quarter..</td>
<td>0 4 12</td>
<td>0 4 12</td>
<td>0 4 12</td>
</tr>
</tbody>
</table>

See Table 3 for definitions.
The Mundell effect is present: the faster the adjustment of prices, the larger the increase in the initial interest rate. The impact effect of the decrease in money is however smaller, the faster prices adjust. The reason for both results in this model is clear: faster price adjustment indeed means higher real rates initially, but lower nominal and real rates later as real money balances increase faster; the faster the price adjustment, the lower the increase in long real rates. Consumption, through wealth, and investment, through \( q \), depend mostly on long real rates. The short-term rate has only one direct effect, given wealth and \( q \), the effect of bending the path of consumption (equation \([1.14]\)) thus inducing consumers to postpone consumption temporarily. The simulations suggest that this effect, although present, is not very strong. Thus, in our model, faster price adjustment leads to a smaller and shorter recession, contrary to Tobin's analysis \(^{13}\).

Note that the long period of high unemployment leads to lower real wages. In all three cases, these lower real wages require a period of overemployment: after the initial recession, the economy experiences a temporary boom, starting in quarter 8 in the first case, in quarter 16 in the second, and quarter 6 in the third. The boom is a Keynesian boom, with aggregate demand below supply. The economy thereafter returns to equilibrium. We now turn to the speed of real wage adjustment.

2.6. Simulation 4. Responsiveness of real wages and Keynesian unemployment

As emphasized by many authors, a decrease in the real wage is likely to improve the economy in a classical regime but may well have perverse effects in a Keynesian regime. We now consider the effects of real wage responsiveness in such a case. Table 6 reports the effects of the decrease in money, for three different values of \( \theta \), 0.005, 0.015 and 0.045.

Because of our formalization of consumers and our assumption of no liquidity constraints, the distribution of output between profits and real wages has no income distribution effect on consumption. Real wages however have an effect on investment: for an output constrained firm, if real wages are anticipated to be lower for a sufficiently long period of time, the firm will aim at a lower capital/labor ratio and thus further decrease investment. The result will therefore be a further decrease in aggregate demand.

This impact effect is there in table 6. Large responsiveness of real wages leads to a larger decrease in investment and aggregate demand; the difference is however small across values of \( \theta \).

---

12. The Mundell effect is currently felt in the U.S. where as the result of lower money growth, high nominal rates and lower inflation have led to short-term real rates around 8%.

13. To reverse these results and obtain the Tobin result, the short-term rate must have a strong impact on economic activity. This may be the case if, for example, there are institutional restrictions in financial markets, leading to rationing of specific sectors such as housing, when short rates increase.
Of more interest is the process of adjustment which is not monotonic but cyclical, due to the interaction of investment, output and real wages. The initial period of recession and low investment is followed by a period of expansion and higher investment. This is particularly clear when real wages are very responsive. During this adjustment, aggregate demand is sometimes larger than aggregate supply, as in column 6 and the economy oscillates between Keynesian and classical regimes.

**Table 6**

**Real wage responsiveness and Keynesian unemployment**

<table>
<thead>
<tr>
<th></th>
<th>Reference case ((\theta = 0.015))</th>
<th>Large responsiveness ((\theta = 0.045))</th>
<th>Small responsiveness ((\theta = 0.005))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter...</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Variable</td>
<td>C</td>
<td>q</td>
<td>r</td>
</tr>
<tr>
<td>C.</td>
<td>(-5)</td>
<td>(-6.6)</td>
<td>(-2.4)</td>
</tr>
<tr>
<td>q...</td>
<td>(-1.8)</td>
<td>(-7.6)</td>
<td>(-8.9)</td>
</tr>
<tr>
<td>QD...</td>
<td>(-9.3)</td>
<td>(-9.7)</td>
<td>(-9.7)</td>
</tr>
<tr>
<td>QS...</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>n...</td>
<td>3.3</td>
<td>4.3</td>
<td>4.3</td>
</tr>
<tr>
<td>a...</td>
<td>11.6</td>
<td>12.1</td>
<td>11.1</td>
</tr>
<tr>
<td>P...</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W/P...</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

See table 3 for definitions.

We now turn to the effects of an anticipated profit tax.

3 **Anticipations of a profit tax**

The accession of a socialist government obviously creates large changes in anticipations. Among those changes, two are likely to complicate the task of economic policy initially. The first is the anticipation of large budget deficits in the future, partially monetized and leading to higher inflation. The second is the anticipation of lower profits by firms, either because of higher real wages and compensation, or higher taxation of firms (see Kolm [15]). Although these anticipations may, in the case of France, not be warranted, real and financial decisions based on them will affect interest rates, the stock market, exchange rates as well as investment, consumption and so on. An understanding of these anticipation effects is important for the conduct of economic policy. In this section, we focus on the effects of the anticipations of lower profits, summarizing the anticipations of various changes in the tax structure by an anticipation of a higher tax rate on profits.
3.1. Taxes and the government budget constraint

For our purpose, we need to introduce only two taxes. The first is a tax at rate $\tau$ on profit $Q - (W/P)L$. The second is a lump-sum tax or subsidy on income. The government budget constraint is:

$$\tau(Q - (W/P) L) + T = 0$$

An increase in $\tau$ implies a corresponding decrease in $T$; thus a higher profit tax does not, ceteris paribus, affect the income of consumers but only its composition.

The presence of a profit tax modifies two of the equations of Table 1 in Section I. The terms $(W/P) (F_K (K, L)/F_L (K, L))$ in equation (1.13) and $(Q - (W/P)L)$ in equation (1.19) are now premultiplied by $(1 - \tau)$.

3.2. The effects of an actual profit tax

We first consider the effects of the actual implementation of an increase in $\tau$, from 0 to 0.2, unanticipated and permanent. The effects are summarized in Table 7.

Two elements explain the results of Table 7. The first is the long-run elasticity of savings with respect to interest rates; the second, the degree of wage responsiveness to unemployment.

The first is readily understood by looking at the steady state effects of the profit tax. In steady state, the interest rate has to remain equal to the subjective discount rate. As $q^*$ is also unchanged, the after-tax marginal product of capital is invariant to changes in the tax rate. In steady state, therefore, the tax is entirely shifted from capital to labor. The decrease in the capital stock depends on the elasticity of substitution between capital and labor. For the assumed value of 0.5 for $\sigma_{KL}$, the capital stock decreases by 14% and so does the capital labor ratio. The real wage is then lower by 8% and output lower by 4%.

The degree of wage response to unemployment determines the dynamics of adjustment, particularly the speed at which the tax is shifted from capital to labor. With flexible wages, as the capital decreases and full employment is maintained, wages decrease, the after-tax marginal product increases and $q$ returns to its steady state value. If real wages do not adjust, however, as the capital decreases employment too decreases. The after-tax marginal product remains the same and $q$ remains below its steady state value. In the extreme case of permanently fixed real wages, this conflict about income distribution leads to complete capital decumulation over time, and zero employment. If real wages respond to unemployment, the economy goes through a protracted period of unemployment and capital decumulation.

---

14. The result that the interest rate remains unchanged and that the tax is ultimately shifted to labor follows from the assumption that agents are—or act as if they were—infinitely lived. If agents have finite horizons, the tax would only be partially shifted to labor. If, however, the economy is small and open and there are no restrictions on capital movements, the steady-state interest rate would again equal the foreign interest rate and the tax would be fully shifted to labor.
Table 7 shows that the adjustment takes the economy through two regimes. The imposition of the tax leads to a decrease in investment demand and a period of Keynesian unemployment (there are two conflicting effects on consumption: the first is the anticipation of lower income, the second the anticipation of constraints on future consumption spending. In the two cases considered, the anticipatory buying effect dominates). As capital decu-

### Table 7

**Profit tax and real wage responsiveness**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Reference case (θ = 0.015)</th>
<th>Small responsiveness of wages (θ = 0.005)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Quarter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.........</td>
<td>1.4</td>
<td>2.2</td>
</tr>
<tr>
<td>q.........</td>
<td>-11.1</td>
<td>-5.1</td>
</tr>
<tr>
<td>QD........</td>
<td>-6.7</td>
<td>-1.1</td>
</tr>
<tr>
<td>QS.........</td>
<td>0</td>
<td>-2.3</td>
</tr>
<tr>
<td>K.........</td>
<td>0</td>
<td>-5.1</td>
</tr>
<tr>
<td>r.........</td>
<td>2.8</td>
<td>0.8</td>
</tr>
<tr>
<td>u.........</td>
<td>8.3</td>
<td>0.9</td>
</tr>
<tr>
<td>W/P......</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

See Table 3 for definitions.

### Figure 3

**Effects of an unanticipated profit tax**

The tax is implemented in quarter 0 (1980).
mulates, the economy enters a phase of classical unemployment (after 4 quarters in the first case, 2 quarters in the second), which lasts for more than 30 quarters in the first case, more than 50 in the second case. If the responsiveness of real wages to unemployment is small, capital decumulates below its new steady state level during the adjustment process.

### 3.3. The effects of an anticipated profit tax

Suppose now that the tax, instead of being currently implemented, is anticipated for some time in the future. Table 8 reports the effects of such anticipations. For both simulations, quarter 0 is the quarter in which firms start anticipating the profit tax; this quarter may be the quarter of the elections, or a prior quarter if the outcome of the elections was anticipated. In the first simulation, the profit tax is anticipated for 8 quarters ahead and in the second simulation for 12 quarters ahead. 15,16

**Table 8**

*Effects of an anticipated profit tax*

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Profit tax anticipated for quarter 8</th>
<th>Profit tax anticipated for quarter 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>1  2  3  4</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0  4  8  12</td>
<td>0  4  8  12</td>
</tr>
<tr>
<td>q</td>
<td>0.9  2.1  1.3  -0.9</td>
<td>0.7  1.4  1  0.8</td>
</tr>
<tr>
<td>QD</td>
<td>-5.8 -4.3 -5 -3.2</td>
<td>-4.1 -3 -3.4 -4.4</td>
</tr>
<tr>
<td>QS</td>
<td>-3.6 -1.6 -1.5 -1.6</td>
<td>-2.6 -1.2 -1.2 -1.8</td>
</tr>
<tr>
<td>K</td>
<td>0 -3.1 -5.7 -8</td>
<td>0 -2.2 -4 -5.6</td>
</tr>
<tr>
<td>r</td>
<td>1.6  1.4  0.3 -1.3</td>
<td>1.2  0.9  0.3  0.3</td>
</tr>
<tr>
<td>u</td>
<td>4.3  0.9  1.8  2.3</td>
<td>3  0.7  1.2  1.8</td>
</tr>
<tr>
<td>W/P</td>
<td>0.0 -1.3 -1.8 -2.8</td>
<td>0 -0.9 -1.3 -1.8</td>
</tr>
</tbody>
</table>

*See table 3 for definitions.*

15. Firms are unlikely to have such precise anticipations. They are more likely to think that the profit tax rate may be increased with some probability which is an increasing function of time. We could formalize this by considering the sequence of expected values of their subjective distribution of tax rates. We could not however characterize the effects of uncertainty per se on their behavior.

16. If the government does not actually intend to increase the profit tax, these simulations give only the anticipated future as of quarter 0. If in quarter 8 in the first case (or 12 in the second), there is no increase in the profit tax and agents revise their anticipations, the outcome in quarters 8 and following will be different from table 8.

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The effects are simple. The change in anticipation leads to a substantial decline in investment (and a decrease in the stock market of 7% in the first case, 5% in the second). This decline in investment demand leads to a Keynesian recession. As capacity decreases, the economy goes into classical unemployment even before the profit tax is implemented. The process of adjustment after the implementation is similar to the one described above.17

**FIGURE 4**

**Effects of an Anticiped Profit Tax**

![Graph](attachment:image.png)

The tax is anticipated as of quarter 0 (1980) for quarter 8 (1982).

Thus anticipations of a profit tax lead first to Keynesian unemployment, then, later on, to classical unemployment. If faced with such anticipations, what should be the response of the government? The answer to this question is difficult but the following remarks can be made.

The most obvious strategy, if the government does not plan any increase in profit taxes, is to attempt to modify anticipations by the presentation of a credible set of fiscal policies for the short and medium terms.

17. In this case, excess shadow demand for goods is never very large and thus there is no substantial inflation. It is however quite possible to generate as a result of an adverse supply shock (profit tax, increased price of some input or technological shock), a period of classical unemployment, a decrease in capital accumulation due to the conflict in income distribution and substantial inflation as chronic excess demand remains. Such an outcome may have some explanatory power for what happened in the second half of the 70's.
The second is to use a short-term expansionary policy to avoid the initial recession, while anticipations of a profit tax disappear. This may be done by measures to help either consumption or investment. Helping consumption may not be successful if the economy is already close to classical unemployment. Helping investment may increase both demand and supply. Thus, the best policy may be, ironically, a temporary reduction in the profit tax, which has effects both directly and as a signal of the government’s intentions.

Conclusion

The effect of a policy on the economy depends very much on whether it has been anticipated or not, on how long it is expected to last and so on. The model developed in this paper, which allows for rational firms and households, as well as for imperfectly flexible prices, is well adapted to characterize the effects of policy in such cases.

It is obviously too preliminary to be a reliable guide to policy: The lack of foundations of price inertia, the infinite life assumption which implies no income distribution effects, the closed economy assumption all need to be relaxed. We believe however that, such as it is, it can shed light on the effects of policy.

References


The Blanchard and Sachs article is an important theoretical contribution to the non-market-clearing approach to macroeconomics. Some people dismiss this approach on the a priori ground that it does not supply any theoretical foundations for the assumption of imperfect price flexibility. What the authors do however is to follow this approach without too many apologies and to show that it is a fruitful one.

In my opinion we are not far away with this paper from what may be a canonical model of intertemporal disequilibrium, playing roughly the same role as the one played in a more static framework by the famous three-regime model notably analysed by Barro-Grossman, Benassy and Malinvaud. The crucial features here are the use of Tobin’s $q$ to derive a genuine investment behavior and the perfect foresight assumption. This leads to a tractable model providing many insights and allowing for various applications and developments. The theoretical progress is unmistakable. To achieve full success, I think however that the analytical treatment should be carried a little further. Simulation would probably then also remain necessary, but at least the working of the model should be more thoroughly explored.

I will thus limit my comment to a few formal points. These concern mainly the role of the transversality conditions. I think that these conditions must not be considered merely as a technical issue, or as an auxiliary device to make sure that the model will not explode. The role of these transversality conditions is to determine the initial values of the costate variables $q$ and $x$, and these are important from an economic point of view since it is through them that future conditions, and especially the expected future constraints, affect the present situation. This is particularly clear in the consumer problem: the differential equation governing $x$ is very simple and the important phenomenon — the wealth effect — passes through $x_o$.

Thus I regret in the first place that the transversality condition in the consumer problem clearly has not been written correctly: the condition in the paper does not imply any restriction on the value of $x_o$. Moreover the a priori condition on the evolution of $A$, which is necessary for the problem to be meaningful, is missing. Of course the formulation of transversality conditions in infinite time is still a somewhat open question but there seems to be no doubt, in the present case, that the traditional condition

$$\lim_{t \to +\infty} x e^{-at} A = 0$$

holds.
Let us turn now to the interpretation of figures 1 and 2. I found the related discussion interesting and important but still somewhat frustrating because the method of analysis is not very explicit. This method seems to me related to the following conditions, based on Hayashi, which state an expression for the initial values $q_o$ and $x_o$ deriving from the transversality conditions:

$$q_o k_o = V_o - \int_{t=1}^{t} \lambda_t I \exp \left( - \int r_s ds \right) dt - \int_{Q=Q_0}^{t=Q} \lambda_Q Q \exp \left( - \int r_s ds \right) dt$$

$$x_o (A_o + W_o) = \frac{1}{\delta} - \int \lambda_c C e^{-\delta t} dt$$

$V_o$ stands for the value of the firm and $W_o$ for the discounted sum of wages and profits. The integrals are calculated over the domain where the constraint effectively applies. The second relationship is derived on the basis of a CES utility function (see table 2) and using the corrected transversality condition.

The path described in figure 2 derives from the second preceding formula. When going from an unconstrained situation to a constrained one $x_o$ decreases and thus desired consumption increases. This conforms to intuition; since the consumer's wealth is given, if he must consume less from $t_1$ to $t_2$ he will consume more for the rest of his life.

If the producer expects not to be constrained we have $q_o K_o = V_o$. This is Hayashi's result: marginal $q = \text{average} \ q$. If the producer expects to be constrained, we have to take into account the deadweight loss of future constraints. I must admit that the previous formula does not seem to me to imply the behavior described on figure 1 and I must express my perplexity before this figure (I do not even see in which case the unconstrained producer would choose to maintain a given absolute level of investment...).

Lastly I would have been happy if the paper had dealt with the possible occurrence of an infinity of solutions in a dynamic model with rational expectation. To this end it would be necessary to distinguish clearly between the relationships describing individual behavior and the ones describing market equilibrium. Once again the transversality conditions would play a crucial role.
Comments

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BLANCHARD and SACHS develop an elegant model of an economy that alternates between states of Keynesian and classical unemployment. In a Keynesian state the real money stock is “too low” and firms are constrained by lack of effective demand. In a classical state, the real wage rate is “too high” and buyers are constrained in the output market. BLANCHARD and SACHS assume away a third state of disequilibrium, that of repressed inflation, to simplify their dynamic analysis. One justification for assuming away excess demand in the labor market might be that real wages adjust immediately upwards while they adjust only slowly downwards. BLANCHARD and SACHS, however, simply postulate that labor supply adjusts passively to labor demand.

The novelty of the BLANCHARD and SACHS model is the explicit specification of household and firm behavior in terms of continuous-time intertemporal optimization subject to exogeneously given prices and quantity constraints which may or may not be binding. It is assumed further that households and firms have perfect foresight with respect to the time path of prices and quantity constraints, and that the economy is always in expectation equilibrium although markets fail to clear. One might, of course, ask why economic agents do not incorporate any of the information concerning future excess demand in their pricing decisions. As it is, BLANCHARD and SACHS assume “as if” prices and wages were determined by an outside “price and wage control board” that follows mechanical rules of price adjustment. In particular it adjusts prices in response to realized excess demand in real time rather than in response to conjectural excess demand. Accordingly excess supply or demand can persist for a long time, which is, of course, a feature that BLANCHARD and SACHS want to have in their model.

I would like to note one feature of the price adjustment rule that BLANCHARD and SACHS assume. As it is specified, prices respond to changes in production costs only gradually through changes in excess demand. This contrasts with the standard mark-up price equation and is obviously responsible for some of the dynamic properties of the BLANCHARD-SACHS model. For example, a reduction of nominal wages is not immediately passed on to prices in their model and accordingly may lead to a reduction rather than an increase in investment and aggregate demand.

A central and novel feature of the BLANCHARD-SACHS model is the modelling of capital formation in a way that explicitly incorporates quantity constraints (MALINVAUD has, of course, emphasized capital accumulation from a somewhat different point of view). As BLANCHARD and SACHS demons-
trate, the effects of changes in real wages or in the cost of capital are quite different depending on the current and expected future state of the economy. If firms are constrained by effective demand and expect to be so constrained also in the future, an increase in nominal wages leads to an increase in investment because of capital-labor substitution. It will also lead to an increase in effective demand in the Blanchard-Sachs model because the wage increase is not passed on to prices. Thus paradoxically an increase in wages may increase employment in the short run. The model would seem to imply that labor will benefit also in the medium term because the supply of capital relative to labor will increase. Since Blanchard and Sachs do not distinguish between capacity-increasing and labor-saving capital, investment is always good for labor. Even if firms invest today to save labor, they can convert the additional capital to higher capacity with higher employment and lower productivity in the future. This is obviously not possible if one takes a richer, vintage view of capital.

Blanchard’s and Sachs’ discussion of the Mundell effect is interesting and relevant: is it possible as Tobin has suggested that rapid price adjustment makes recessions more severe because of the effect of anticipated price deflation on relative asset yields? Simulation of the Blanchard-Sachs model suggests that this is not the case: the faster prices adjust, the less is the effect of monetary contraction on unemployment. This result might not hold, however, if one introduced liquidity trap considerations or the possibility of bankruptcies and shifts in liquidity preference in the process of severe price deflation.

The simulation exercises that Blanchard and Sachs carry out in the last few pages of their paper concerning actual and anticipated profits taxes illuminate the usefulness of their model in tracing out the dynamic effects of policy changes. In their model a profits tax is completely shifted to wages in the long run because the supply of capital is assumed to be infinitely elastic at the subjective rate of time preference. In the short run, the effect of a profit tax depends crucially on the state of the economy as well as on future expectations.

A profits tax increases the cost of capital and therefore causes firms to substitute labour for capital. In the simulations that are reported this positive effect on employment is, however, more than offset by the depressing effect of lower investment on aggregate demand. The economy enters a period of Keynesian unemployment. As the supply of capital is reduced the economy moves on to a state of classical unemployment. If the real wage rate does not adjust, capital decumulation proceeds without limit. In the long run, technology and the cost of capital determine the equilibrium real wage rate. In other words, long run demand for labor is infinitely elastic. This is a strong implication of the model, and it obviously depends on the rather special assumptions that the authors make about saving behavior and technology.

In summary, Blanchard and Sachs have provided us with a useful tool for analysing dynamic adjustment of the economy to changes in policy. This issue is of great importance, for the feasibility of policies that are desirable from the long run point of view often depends on their short term and medium term effects.